On the effect of a co-flowing stream on the structure of an axisymmetric turbulent jet

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Abstract

The aim of this paper is to study experimentally the turbulent structure of an axisymmetric jet submerged in the co-flowing stream of the test section of a wind tunnel. The experiments have been performed without and with three different velocities for the outer stream. The measurements were carried out by using Pitot tube and Stationary Hot-Wires. The influence of the outer stream and the confinement effect on the development of the turbulent jet are analyzed. It is shown that the streamwise variation of the excess mean velocity, the jet expansion and the spanwise distribution of the longitudinal normal Reynolds stress are close to that observed on free jets by other research groups. Self-similarity is also investigated. An extensive spectral analysis has also been done, and the centerline variations of the integral time scales and Taylor microscales are also presented. Only the third-order moment of the longitudinal velocity fluctuation exhibits a radial distribution very different from that expected in free jets. In the case of no outer stream, it is shown that the confinement effects dominate and modify significantly the jet structure.

Keywords: Turbulence measurements; Axisymmetric jet; Co-flowing stream; Stationary hot-wire

1. Introduction

A large number of studies have been devoted to axisymmetric free jets such as for instance that of Wygnansky and Fiedler [1] and more recently those of Panchapakesan and Lumley [2] and Hussein et al. [3]. Relying on the experience gained from previous works, the two latter have set experimental conditions where the difficulties introduced by large turbulent intensities and the confinement of the jet are fully controlled and taken into account. The flows hence obtained may be considered as real free jets. Their results have been obtained by means of special anemometric techniques such as “Flying Hot-Wires” (FHW) or Laser Doppler Anemometry (LDA). Pioneering studies have also been conducted on coaxial jets [4] or on round jets emitted in a uniform co-flow (e.g. [5]). The experimental problems related to high level turbulent flows can in part be discarded by superimposing a mean free stream to the jet as fulfilled by Antonia and Bilger [6] or more recently by Sreenivasan et al. [7]. It is this latter configuration which is retained in the present work. The aim is to put in evidence the influence of an external mean velocity and confinement effects on the mean and turbulent structure of the jet flow. The study will concentrate on the developed region, with $30 < x/D < 120$. For different co-flow conditions, the evolution of the longitudinal mean velocity on the axis, the rate of expansion of the jet, the Reynolds stresses and the triple correlations were investigated. Further, spectral analysis enabled the calculation of integral and microscales.

2. Experimental setup

The air jet is issued from a cylindrical nozzle with an external diameter of 8 mm and an internal diameter of 4 mm, built according to a sketch by Rizk and Lefebvre [8] and placed along the axis of a rectangular square test section ($20 \times 20$ cm$^2$) of a wind tunnel. The jet develops therefore in a confined space and it is possible to superimpose a known mean stream by adjusting the wind tun-
nel flow rate (Fig. 1). The test section is preceded by a 10:1 contraction, the boundary layer of the co-flow is fully turbulent and is 20 mm thick, the center region stream without the injector presenting a turbulence level of less than 1%.

In the case of the present experiment, the outlet velocity of the jet was set to \( U_1 = 180 \text{ m/s} \) and the outer stream velocity to \( U_1 = 6.00 \text{ m/s} \) (case 1). However, in order to underline certain features of the flow, the latter has also been adjusted to 3.31 m/s (case 2) and 8.39 m/s (case 3). The jet Reynolds number is defined as

\[
\text{Re} = \frac{U_1 D}{v},
\]

it is of the order of 48 000. The various parameters and experimental conditions are summarized in Table 1. The turbulent jet without co-flow was also investigated and is referenced as case 4.

The mean velocity field has been measured using a standard Pitot tube and a Furness differential manometer with an uncertainty of water 0.005 mm yielding a relative uncertainty of \( \pm 1\% \), at the 95% confidence level.

Mean velocity and turbulence measurements (one or two components) were further performed with identical Dantec hot-wire anemometers, each including a control unit (55M01), a signal conditioner (55D10) and a linearizer (55D26). These two analogic processing units had the same settings in order to avoid as much as possible the effect of a phase shift on correlation measurements made with the X-wire. The length and the diameter of the sensor filaments were, respectively, 1.25 mm and 5 \( \mu \text{m} \) (Dantec normal wire 55P01 and standard X-wire probe 55P61 with a \( 90^\circ \) cross angle). The hot-wire probes, operating at an overheat ratio of 1.8, were systematically calibrated before starting a measurement session and, in order to check for possible temperature deviations of the air jet, the calibration was verified again after the session to ensure its validity. The signal outputs from the linearizers were filtered in the 0.1 Hz–10 kHz frequency band and digitized at 25.6 kHz (Nyquist frequency) with a 12 bits resolution using a 2622 Tektronix Fourier analyzer operating in Boxcar mode. The linearized signals were amplified and the gain of the Fourier analyzer optimized for a maximum resolution. As an example, at cross section \( x/D = 36.5 \), the full-range was 24 m/s, with no fluctuation out of the \( \pm 12 \text{ m/s} \) range (RMS value of about 6 m/s at that point), so that the relative resolution was 0.6% (24/212).

Signals were stored on a hard disk for subsequent treatment and were directly Fourier transformed so as to obtain readily the spectra and correlations data which were also stored. For the raw signals, at each measurement point, 500 (for single wires) or 250 (for each cross-wire) data acquisition blocks containing each 2048 samples were registered. The total number of samples collected in both cases ensured a good statistical stability.

The second-order moments (Reynolds stresses) of the fluctuating velocity components were directly obtained by specific data delivered by the Tektronix Fourier analyzer (based on the integration of the spectral energy, see for instance Gibson [9]) and it has been checked that the computed results were in best agreement (less than \( \pm 1\% \) difference) with those yielded by classical integration methods using analogical (Dantec) or numerical voltmeters (Keithley 194A) or those calculated directly from the stored digitalized signals. The variances were corrected according to the classical relations of Champagne et al. [10]. The turbulence intensity was, in most cases (1, 2 and 3), in a not very critical range because of the presence of the outer co-flow so that corrections for instance proposed by Tutu and Chevray [11] or Rodi [12], considering the influence of the transverse mean velocity component or high turbulence level, were not necessary. Case 4, without co-flowing stream, would have needed a total rearrangement of the experimental procedure, involving an improved treatment of the non-linear hot-wire response and an adapted correction technique for high turbulence levels.

The third-moments were directly computed from the linearized outputs as digitalized and stored without any correction scheme. The turbulence measurements, with the careful calibration procedure and cross-checks performed to insure that no temperature drift would occur during a measurement session, were given with a total uncertainty of \( \pm 10\% \) for the Reynolds stresses and \( \pm 15\% \) for the third-moments.

The microscales were computed according to Taylor’s definition, based on the second derivative of the autocorrelation curve. The integral scales were deduced from the autocorrelation curve as well, by a numerical quadrature computing the integral of the correlation function over the time interval \( [0,t_f] \), \( t_f \) being the first zero of the function. The truncation error is negligible, compared to errors induced by omitting possible negative loops of the correlation functions often present for the transverse components, so it is clear that estimating an uncertainty for these integral scales remains difficult.

Table 1
Flow conditions for all experiments

<table>
<thead>
<tr>
<th>( U_1 )</th>
<th>180 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_1 ) (case 1)</td>
<td>6.00 m/s</td>
</tr>
<tr>
<td>( U_1 ) (cases 2, 3 and 4)</td>
<td>3.31, 8.39 and 0 m/s</td>
</tr>
<tr>
<td>Re</td>
<td>48 000</td>
</tr>
<tr>
<td>( D )</td>
<td>4 mm</td>
</tr>
<tr>
<td>( x/D )</td>
<td>36.5, 50.5, 65.0, 85.0, 95.0 and 114.5</td>
</tr>
</tbody>
</table>
3. Variation of the centerline velocity

If the flow is assumed to be self-similar, then the mean velocity $\bar{U}$ can be written as:

$$\bar{U}(x, \eta) = U_1 + U_0(x)f(\eta),$$

(2)

where $U_0(x)$ is the excess centerline velocity, $f(\eta)$ is the similarity function and $\eta = r/\delta(x)$ is the reduced radial coordinate, so that for $r = \delta(x)$:

$$\bar{U}(x, 1) = U_1 + \frac{1}{2} U_0(x).$$

(3)

Fig. 2 yields the longitudinal variation of the ratio of the outlet velocity of the jet $U_1$ to the mean flow velocity on the axis $U_0(x)$ for the four co-flow configurations.

It appears that for cases 1, 2 and 3 (with an external co-flow), the longitudinal variation of the reciprocal of the excess centerline velocity, $U_1/U_0$, plotted versus the normalized distance to the jet outlet $x/D$, approaches a linear law. This feature is not very different from the experimental data found by other authors [3,2]. The present result is also consistent with the variation deduced from similarity considerations.

Using the notations of Hussein et al. [3] one can write

$$\frac{U_1}{U_0(x)} = \frac{1}{B_u} \frac{x - x_0}{D},$$

(4)

where $x_0$ is a virtual origin.

The values of $B_u$ and $x_0$ for the various cases are summarized in Table 2. Comparison with previous results obtained by various authors shows that the mean velocity on the axis of the jet decays more slowly when the co-flow is higher. Without an outer flow, the reciprocal of the mean velocity on the axis first obeys a linear variation, but linearity breaks down in the far field, for $x/D$ larger than 90. This non-linear evolution is identical to that observed by Wygnanski and Fiedler [1] and to that described by George [13] and Hussein et al. [3] for a confined jet, which is clearly the case of flow 4. The value of the slope factor $B_u$ so obtained is very close to that given by Wygnanski and Fiedler [1].

4. Mean velocity profiles

Fig. 3 shows that the reduced mean radial profiles of the excess velocity at six sections are self-similar, the radial coordinate $r$ has simply been replaced by $y$, the spanwise distance to the axis, which has been explored for negative and positive values. The symmetric spanwise distribution can be well represented by the expression

$$f(\eta) = \exp(-A\eta^2),$$

(5)

where $A = 0.693 \pm 0.003$ which is not very different from the expected value $\ln 2$.

Self-similar profiles are also preserved for other values of the outer co-flowing stream velocity as shown in Fig. 4. In order to compare the measured evolution of the width of the jet to that obtained by other authors, the normalized velocity profiles $\bar{U}(x, \eta)/U_1(x)$, with $U_1(x) = U_1 + U_0(x)$, are plotted as a function of the transverse coordinate $y$ non-dimensionalized by the longitudinal coordinate $x$ and the new variable $y/x$ is again designed by $\eta$. Due to the similarity, the rate of expansion of the jet is given by the non-dimensional half-

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Table 2

<table>
<thead>
<tr>
<th>Cases</th>
<th>$B_u$</th>
<th>$x_0/D$</th>
<th>$\eta_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.29 ± 0.04</td>
<td>2.6 ± 0.5</td>
<td>0.099 ± 0.001</td>
</tr>
<tr>
<td>2</td>
<td>6.53 ± 0.04</td>
<td>2.0 ± 0.5</td>
<td>0.099 ± 0.001</td>
</tr>
<tr>
<td>3</td>
<td>6.26 ± 0.04</td>
<td>2.4 ± 0.6</td>
<td>0.099 ± 0.001</td>
</tr>
<tr>
<td>4</td>
<td>5.34 ± 0.05</td>
<td>8.9 ± 0.6</td>
<td>0.092 ± 0.001</td>
</tr>
<tr>
<td>Wygnanski and Fiedler [1]</td>
<td>5.4</td>
<td>5</td>
<td>0.086</td>
</tr>
<tr>
<td>Panchapakesan and Lumley [2]</td>
<td>6.04</td>
<td>0</td>
<td>0.096</td>
</tr>
<tr>
<td>Hussein et al. [3] (FHW)</td>
<td>5.8</td>
<td>4</td>
<td>0.094</td>
</tr>
<tr>
<td>Hussein et al. [3] (SHW)</td>
<td>5.9</td>
<td>2.7</td>
<td>0.102</td>
</tr>
<tr>
<td>Rodi [12] (SHW)</td>
<td>5.83</td>
<td>4</td>
<td>0.086</td>
</tr>
</tbody>
</table>
width \( \eta_{1/2} \), so that \( \overline{U}(x, \eta_{1/2})/U_0(x) = \frac{1}{4} \). The results are listed in Table 2. Although the operating conditions are different, our results are comparable to those of Hussein et al. [3] and Panchapakesan and Lumley [2].

However, some differences exist which may be attributed to the various experimental methods used and to the intrinsic nature of the flow configurations. Indeed, according to the analysis of George [13] and Hussein et al. [3], it is clear that the data obtained by stationary hot-wires and by Pitot tubes yield overestimated values of \( \eta_{1/2} \). This certainly explains the discrepancies observed with those obtained by LDA or FHW. Moreover, comparison of the Wygnanski and Fiedler [1] experiment (where a stationary hot-wire was used) with that of Hussein et al. [3] (performed with a SHW), shows the same discrepancy as that observed in our configuration between the case where there is an outer flow and that where there is not. In fact, in case 4, the effect of confinement is predominant and hinders the natural spreading of the jet. It leads to a non-linear evolution of \( U_j/U_0(x) \) and to lower values of \( \eta_{1/2} \). Conversely, when an outer co-flow is superimposed to the jet, the variations are analogous to that observed in free jets and the mean velocity profiles are not altered by the confinement since identical values of \( \eta_{1/2} \) are obtained for different values of the co-flow \( U_1 \). This suggests that the external velocity merely sets in motion the fluid downstream of the injector without influencing the development of the mean flow.

5. Turbulent intensities

The turbulent intensities have been scaled by the maximum mean flow velocity \( U_0(x) \). The RMS value of the fluctuating quantities will henceforth be denoted, both in the text and in the figures, by \( \text{bold characters} \).

5.1. Longitudinal variation of the turbulent intensities

The streamwise distribution of the longitudinal turbulent intensity for the three values of the external velocities are plotted in Fig. 5 with the corresponding evolution for the case without co-flowing stream \( (U_1 = 0) \). The former figure shows that for each value

![Fig. 3. Profiles of the non-dimensionalized longitudinal mean jet velocity at six different streamwise positions (see Table 1 for case 1 \((U_1 = 6.00 \text{ m/s})\).](attachment:image1)

![Fig. 4. Self-similar profiles of the excess mean velocity for cases 2 and 3 \((\text{\&}U_1 = 3.31 \text{ m/s}; +, \times: U_1 = 8.39 \text{ m/s})\).](attachment:image2)

![Fig. 5. Longitudinal variation of the streamwise normal Reynolds stress \( u \) with a co-flowing air flow \((U_1 \neq 0 \text{ m/s})\) and without outer flow \((U_1 = 0 \text{ m/s})\).](attachment:image3)
of the external velocity the non-dimensional turbulent intensity increases slowly and tends to a constant value which is reached when \( x/D \geq 70 \). This tendency has already been observed by Wygnanski and Fiedler [1] and Panchapakesan and Lumley [2]. However, in the case of [1], the "plateau" is obtained for \( x/D = 40 \) which is due to the fact that, in this particular experiment, the jet Reynolds number is 10 times larger than that of the present case. Since the outer flow does not alter significantly the turbulence structure of the jet, the trend of the three curves corresponding to non-zero values of the outer parallel stream velocity are quite similar. One will note that the relative values of the turbulent intensities is lower with larger outer flow velocity. Without external flow \( (U_1 = 0) \), an exponential evolution of \( u/U_1(x) \) is observed. A similar behavior was already described by So et al. [14] in a confined jet without co-flow.

Smaller values of the turbulent intensities, as compared to those obtained by Wygnanski and Fiedler [1] and Hussein et al. [3], are measured in the present work, this is due to the somewhat smaller jet Reynolds number and to the fact that the velocity, used here to normalize the data, takes into account \( U_1 \) in the computation of the intensity \( u/U_1(x) \).

### 5.2. Transverse distribution of the turbulent intensities

The transverse distribution of the turbulent intensity \( u/U_1(x) \) is given in Fig. 6. The coordinate \( y \) is scaled by the profile half-width \( \delta(x) \) (see Eq. (3)). One can observe that similarity is progressively reached as one moves downstream. It is worth noting that the double maximum of the curves have already been observed by Panchapakesan and Lumley [2] and Hussein et al. [3] who used FHW. However Wygnanski and Fiedler [1] and Hussein et al. [3], who used stationary hot-wire techniques, as we have done, did not observe this feature. It seems that this could be due to the fact that the level of the turbulent intensity appears to be smaller when the FHW technique is used or when a mean velocity is superimposed to the flow. Other explanations could refer to jet instabilities.

### 6. Moments of order three

A description of the diffusive terms (\( u\dot{q} \) and \( v\dot{q} \)), present in the equation of conservation of the kinetic turbulent energy is given here. Out of the nine terms that can be expressed, only five have been measured. They have been scaled by \( U_3^3(x) \). All the data show that, as regards to the longitudinal turbulent intensity, self-similarity is obtained for the farthest sections and that their maximum values are reached in the area where the intermittency factor is larger than 0.5 [15].

The variation of \( uu^2 \) in Fig. 7 displays a negative region of the curve near the centerline. This behavior is very different from that observed by Panchapakesan and Lumley [2] and Hussein et al. [3] in the case of free jets. However, Antonia et al. [16] noted a variation similar to ours in the case of submerged jets.

The other triple moments have more standard behaviors. \( uv^2 \) and \( uw^2 \), Figs. 8 and 9, exhibit two extrema and have a general trend comparable to that of \( uu^2 \). Moreover, the values obtained for these extrema (for \( uv^2 \)) are approximately the same as that measured by Panchapakesan and Lumley [2] in a free jet.

This remark also applies to \( vv^2 \) and \( vu^2 \) where the evolution, as shown in Figs. 10 and 11, is the same as that found by Panchapakesan and Lumley [2] and Hus-
sein et al. [3]. In particular, $\overline{uv}$ is an odd function and has a minimum offset from the centerline. Therefore $\overline{uv}$ is the only term which behaves differently from what observed in free jets.

As to the turbulence intensities, one must add that, with identical experimental techniques, the profiles for $\overline{uv}^2$, $\overline{uw}^2$ and $\overline{vu}^2$ given by Wygnanski and Fiedler [1] and Hussein et al. [3] (SHW) do not exhibit the negative values which are obtained when the LDA or FHW techniques are used [3].

7. Turbulent scales

The integral scales have been computed by integrating the autocorrelation functions

$$T_{d0} = \int_0^{t_f} R_{uw}(t) \, dt, \quad (6)$$

where $T_{d0}$ denotes the longitudinal time integral scale measured on the centerline of the flow, $R_{uw}(t)$ the longi-
Longitudinal autocorrelation function and \( t_f \) the first zero of the autocorrelation function \( R_{uu}(t) \) [15]. An analog expression can be written for the transverse time integral scale \( T_v \) by replacing \( u \) by \( v \). Both have been non-dimensionalized by \( U_j/D \).

In Fig. 12, the effect of the outer co-flowing stream is analyzed. Indeed, when \( U_1 \) is zero, \( T_u \) follows a power law of exponent 2 otherwise, the variation is linear according to

\[
\frac{\Psi}{D} = A_T + B_T \frac{x}{D},
\]

where \( \Psi = T \) for integral scales and \( \Psi = \tau \) for microscales. Table 3 resumes the values of \( A_T \) and \( B_T \) for the various experimental conditions.

Longitudinal and transverse Taylor microscales, respectively \( \tau_u \) and \( \tau_v \), measured on the centerline of the flow are defined by

\[
\frac{1}{\tau_{uw}} = -\frac{1}{2u^2} \left[ \frac{dv^2 R_{uu}(t)}{dt^2} \right]_{t=0}.
\]

One will note in Fig. 12 that the value becomes larger when the outer velocity decreases. This seems coherent since the eddy convection velocity increases the signal frequencies as sensed by the fixed hot-wire, hence leading to a more rapid decorrelation from which it follows that the values of the time scale \( T_d \) and \( T_v \) diminish.

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\]

Table 3 resumes the values of \( A_T \) and \( B_T \) for the various experimental conditions.

<table>
<thead>
<tr>
<th>Cases</th>
<th>( A_T )</th>
<th>( B_T )</th>
<th>( A_v )</th>
<th>( B_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: ( \psi_\omega )</td>
<td>-8.6 ± 0.6</td>
<td>0.455 ± 0.008</td>
<td>2.42 ± 0.06</td>
<td>(68.8 ± 0.8) \times 10^{-3}</td>
</tr>
<tr>
<td>2: ( \psi_\omega )</td>
<td>-2.2 ± 0.5</td>
<td>0.198 ± 0.006</td>
<td>1.80 ± 0.06</td>
<td>(49.6 ± 0.7) \times 10^{-3}</td>
</tr>
<tr>
<td>3: ( \psi_\omega )</td>
<td>13 ± 1</td>
<td>0.58 ± 0.01</td>
<td>2.07 ± 0.05</td>
<td>(83.5 ± 0.6) \times 10^{-3}</td>
</tr>
<tr>
<td>4: ( \psi_\omega )</td>
<td>5.4 ± 0.4</td>
<td>0.363 ± 0.005</td>
<td>3.0 ± 0.2</td>
<td>(52 ± 2) \times 10^{-3}</td>
</tr>
</tbody>
</table>

In Fig. 13 we can see that in all cases microscales have the same trends that the integral scales. We find a linear evolution for cases 1–3 and a power law of exponent 1.09 ± 0.04 for case 4. Eq. (7) gives \( A_v \) and \( B_v \), these values are summarized in Table 3.

8. Conclusion

The present study has enabled to put in evidence the effect of a velocity co-flowing stream and the effect of confinement on the development of an axisymmetric jet discharging on the centerline of the test section of a wind tunnel. One shows that the variation of the mean velocity and the rate of expansion of the jet is identical to that of a free jet, which suggests that the inner flow (jet) is not affected by the external flow. A similar conclusion can be drawn concerning the turbulent quantities where only \( \psi_{uu} \) exhibits a different behavior near the axis of the flow. In the absence of entrainment velocity, the confinement effect seems to be dominant and the structure of the flow is significantly different from that of a free jet. Moreover, this study shows the influence of high turbulent intensities on the hot-wire measurement (SHW).
Nomenclature

\( \bar{A} \)  Gaussian curve fit for the self-similar function
\( B_p \)  decay rate of the centerline velocity
\( D \)  internal diameter (m)
\( \text{Re} \)  jet Reynolds number
\( R_{uu}(t) \)  temporal autocorrelation function (m\(^2\)/s)
\( T_{10}, T_{01} \)  centerline longitudinal and transversal integral scales (s)
\( \bar{U} \)  mean velocity (m/s)
\( U_0(x) \)  excess centerline velocity (m/s)
\( U_l \)  outer velocity (m/s)
\( U_i \)  injection velocity (m/s)
\( U_m(x) \)  maximum mean velocity (m/s)
\( f(\eta) \)  self-similar function
\( t_i \)  first zero of the autocorrelation function (s)
\( u, v, w \)  axial, transversal and vertical fluctuating velocities (m/s)
\( \bar{q}^2 \)  kinetic energy (m/s\(^2\))
\( x, r \)  longitudinal and radial coordinates (m)
\( x_0 \)  virtual origin (m)
\( \delta \)  half-width of the jet (m)
\( \eta \)  dimensionless radial coordinate
\( \eta_{1/2} \)  dimensionless half-width
\( \psi \)  argument or subscript (Taylor microscale or integral scale)
\( \tau_{10}, \tau_{01} \)  centerline longitudinal and transversal Taylor microscales (s)
\( A_p, B_p \)  linear curve fit for the \( \psi \) function (Taylor microscale or integral scale)
\( \langle \rangle , \langle \rangle \)  time average

References